Technical appendix to the paper "Self-reinforcing effects between housing prices and credit."

André K. Anundsen* Department of Economics, University of Oslo

Eilev S. Jansen Statistics Norway

May 14, 2013

1 The cointegrated VAR model of Section 5 in the paper

With reference to the discussion in Section 5 of the paper, the VAR(5), which is the starting point for our econometric analysis, may be reparameterized in the following way:

$$\Delta \mathbf{x}_t = \mathbf{\Pi} \mathbf{y}_{t-1} + \sum_{i=1}^{4} \mathbf{\Gamma}_i \Delta \mathbf{x}_{t-i} + \sum_{i=0}^{4} \mathbf{\Psi}_i \Delta \mathbf{z}_{t-i} + \mathbf{\Phi} \mathbf{G_t} + \epsilon_t, \tag{1}$$

where $\boldsymbol{\epsilon_t} \sim \mathrm{N}(\mathbf{0}, \mathbf{\Sigma})$, $\mathbf{x}_t$ is a 3×1 vector comprising the endogenous variables ph,d and yh. $\mathbf{y} = (\mathbf{x}', \mathbf{z}')'$ is a $(3+3) \times 1$ vector where $\mathbf{z}$ is a 3×1 vector composed of the weakly exogenous variables R, th and h. $\mathbf{G_t}$ is a vector of deterministic terms (constant, linear trend and centered seasonal dummies), and $\mathbf{\Pi}$, $\mathbf{\Gamma}_i$, and $\mathbf{\Psi}_i$ and $\mathbf{\Phi}$ are the corresponding coefficient matrices.

In the analysis in Section 5 of our paper, we follow the suggestion of Harbo et al. (1998) for partial systems and restrict a deterministic trend to enter the cointegration space. This implies that equation (1) can be written as:

*Corresponding author, PO Box 1095 Blindern, N0317 Oslo, email: a.k.anundsen@econ.uio.no.

$$\Delta \mathbf{x}_t = \tilde{\mathbf{\Pi}}\tilde{\mathbf{y}}_{t-1} + \sum_{i=1}^{4} \mathbf{\Gamma}_i \Delta \mathbf{x}_{t-i} + \sum_{i=0}^{4} \mathbf{\Psi}_i \Delta \mathbf{z}_{t-i} + \tilde{\mathbf{\Phi}}\tilde{\mathbf{G}}_t + \epsilon_t \tag{2}$$

where $\tilde{\mathbf{\Pi}} = (\mathbf{\Pi}, \delta)$ and $\tilde{\mathbf{y}} = (\mathbf{y}', t)'$ with δ representing the vector of trend coefficients. Further, $\tilde{\mathbf{G}}_t$ comprises only a constant and centered seasonal dummies with the corresponding coefficient matrix being given as $\tilde{\mathbf{\Phi}}$.

The trace test for the order of cointegration (Johansen, 1988) can be used to determine the rank of the matrix $\tilde{\mathbf{\Pi}}$, which corresponds to the number of independent linear combinations between the variables that are stationary. We follow Johansen (1988) and define $\tilde{\mathbf{\Pi}} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\beta}$ is a $(p+q+1) \times r$ matrix and $\boldsymbol{\alpha}$ is a $p \times r$ matrix corresponding to the long run coefficients and loading factors respectively. The rank of the $\tilde{\mathbf{\Pi}}$ matrix is denoted by r, while p refers to number of endogenous variables and $q+1$ is the number of exogenous variables (including the deterministic trend, which is restricted to lie in the cointegration space).

Given that the rank of $\tilde{\mathbf{\Pi}}$ is two (as we find in the paper), with three endogenous and three exogenous variables, the cointegrating part of equation (1) takes on the following form:

(3)

$$\boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y} = \begin{pmatrix} \alpha_{1,ph} & \alpha_{1,d} \\ \alpha_{2,ph} & \alpha_{2,d} \\ \alpha_{3,ph} & \alpha_{3,d} \end{pmatrix} \begin{pmatrix} \beta_{ph,1} & \beta_{d,1} & \beta_{yh,1} & \beta_{R,1} & \beta_{th,1} & \beta_{h,1} & \beta_{t,1} \\ \beta_{ph,2} & \beta_{d,2} & \beta_{yh,2} & \beta_{R,2} & \beta_{th,2} & \beta_{h,2} & \beta_{t,2} \end{pmatrix} \begin{pmatrix} ph \\ d \\ yh \\ R \\ th \\ h \\ t \end{pmatrix}$$

Exact identification can be achieved by imposing two restrictions in each vector. We start by normalizing on real housing prices in the first vector ($\beta_{ph,1} = 1$) and real household debt in the other ($\beta_{d,2} = 1$). In addition, it is assumed that the housing turnover has no direct effect on real housing prices ($\beta_{th,1} = 0$). Tests for overidentifying restrictions are reported in the paper.

2 The Dynamic model of Section 6 in the paper

To derive the simultaneous equation system that forms the basis for the econometric analysis in Section 6 of the paper, we have premultiplied the reduced form representation in equation (2) by the (non-zero) contemporaneous feedback matrix, $\mathbf{B}$:

$$\text{(4)} \qquad \mathbf{B}\Delta\mathbf{x}_t = \mathbf{B}\tilde{\mathbf{\Pi}}\tilde{\mathbf{y}}_{t-1} + \sum_{i=1}^{4}\mathbf{B}\mathbf{\Gamma}_i\Delta\mathbf{x}_{t-i} + \sum_{i=0}^{4}\mathbf{B}\mathbf{\Psi}_i\Delta\mathbf{z}_{t-i} + \mathbf{B}\boldsymbol{\epsilon}_t$$

where we now define $\mathbf{B}\tilde{\mathbf{\Pi}} = \boldsymbol{B\alpha\beta'} = \boldsymbol{\alpha}^*\boldsymbol{\beta}', \boldsymbol{B}\mathbf{\Gamma}_i = \mathbf{\Gamma}_i^*, \boldsymbol{B}\mathbf{\Psi}_i = \mathbf{\Psi}_i^*, \boldsymbol{B\epsilon}_t = \boldsymbol{\varepsilon}_t$. In the interest of expositional simplicity, we have left out the deterministic terms, $\tilde{\mathbf{G}}_t$, from the equation. The new error term will also be IIN with zero mean and variance-covariance matrix given by: $\mathbf{\Omega} = E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \mathbf{B}E(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t')\mathbf{B}' = \mathbf{B\Sigma B}'$.

As the income variable is found to be weakly exogenous, we can write the above system as a conditional system for housing prices and credit and a marginal model for income (see e.g Johansen (1992)). Since the focus of our paper is the interaction between housing prices and credit, we can, without loss of generality, abstract from modeling the marginal model for income. In that case, the conditional SVECM takes the following form:

$$\text{(5)} \qquad \Delta ph_t - b_{12}\Delta d_t = \sum_{i=1}^{4}\mathbf{\Gamma}_{1i}^*\Delta\mathbf{x}_{t-i}^* + \sum_{i=0}^{4}\mathbf{\Psi}_{1i}^*\Delta\mathbf{z}_{t-i}^* + \sum_{i=1}^{4}\widetilde{\Psi}_{1,Ri}\Delta R_{t-i} + \alpha_{1,ph}^*ECM_{t-1}^{ph} + \alpha_{1,d}^*ECM_{t-1}^{d} + \varepsilon_{ph,t}$$

$$\text{(6)} \qquad -b_{21}\Delta ph_t + \Delta d_t = \sum_{i=1}^{4}\mathbf{\Gamma}_{2i}^*\Delta\mathbf{x}_{t-i}^* + \sum_{i=0}^{4}\mathbf{\Psi}_{2i}^*\Delta\mathbf{z}_{t-i}^* + \sum_{i=1}^{4}\widetilde{\Psi}_{2,Ri}\Delta R_{t-i} + \alpha_{2,ph}^*ECM_{t-1}^{ph} + \alpha_{2,d}^*ECM_{t-1}^{d} + \varepsilon_{d,t}$$

where we have normalized such that the contemporaneous feedback matrix, $\mathbf{B}$, has ones along the main diagonal. $\mathbf{x}^*$ now consists of the two remaining endogenous variables, while $\mathbf{z}^*$ still represents a vector of the current and lagged exogenous variables in the system (including the income variable) as well as a constant and centered seasonal dummies. $\mathbf{\Gamma}_{ji}^*$, $\mathbf{\Psi}_{ji}^*$ and $\widetilde{\Psi}_{j,Ri}$ (j=1, 2) are the short run coefficients, where $\mathbf{\Gamma}_i^* = (\mathbf{\Gamma}_{1i}^*, \mathbf{\Gamma}_{2i}^*)$ and $\mathbf{\Psi}_i^* = (\mathbf{\Psi}_{1i}^*, \mathbf{\Psi}_{2i}^*)$. Since the housing stock adjusts slowly, it is assumed to be fixed in the short run and is not part of the vector $\mathbf{z}^*$. Note also that we have excluded the contemporaneous value of the change in real after-tax interest rate, ΔR_t, from both equations to form our general unrestricted model. However, we supplement the short run dynamics by including an expectations variable, E, which measures households expectations about future developments in their personal economy and the macroeconomy. Hence, $\mathbf{z}^*= (th, E, yh)$. This is the system that constitutes the general unrestricted model. In Section 6 of the paper, we have reduced the dimensionality of this system by going general to specific. The final model (preferred specification) is reported in Table 5 of the paper.

3 Equation-by-equation modelling

Adopting a single equation approach one would take the system represented by equation (5) and (6) as a starting point. This approach precludes any formal treatment of identification, but may possibly give reasonable results if the simultaneity bias is not large. We have used the automated multipath search algorithm Autometrics (see Doornik (2009) and Doornik and Hendry (2009a)) to reduce the dimensionality of each equation. An obvious advantage with this algorithm is that it is very little path dependent as it does a multipath search. However, the benefit from this might be outweighed by the fact that it does not allow us to take care of the simultaneity from the onset by doing a full fledged system analysis at each step in the reduction process. The results from this single equation general to specific approach are documented in Table 1 and Table 2 for the housing price and credit equation, respectively.

Table 1: Short run dynamics obtained by Autometrics for housing price equation[a]

Variable	Coefficient	t-value
$Constant$	1.23	6.78
Δd	0.61	3.85
Δph_{t-4}	0.41	4.93
Δt_{t-3}	0.05	2.55
Δr_{t-4}	-0.38	2.06
ΔE_t	0.095	4.54
ΔE_{t-1}	0.096	4.40
ΔE_{t-2}	0.05	2.17
ecm^{ph}_{t-1}	-0.07	3.81
ecm^{d}_{t-1}	-0.14	6.80
$CSeasonal_t$	-0.006	0.496
$CSeasonal_{t-1}$	-0.007	0.65
$CSeasonal_{t-2}$	-0.009	0.999
σ	0.0141	
R^2	0.82	
$Adj.R^2$	0.80	
Diagnostics[b]	Test statistic	Value [p-value]
AR 1-5 test:	$F(5,73) =$	0.4789 [0.7909]
ARCH 1-4 test:	$F(4,83) =$	0.4462 [0.7749]
Normality test:	$\chi^2(2) =$	1.5603 [0.4583]
Hetero test:	$F(21,69) =$	1.3658 [0.1672]
Estimation Method	OLS (Autometrics with p-value = 0.05)	
Sample	1986q2-2008q4	

[a] Absolute t-values are reported.
[b] See Doornik and Hendry (2009a).

Table 2: Short run dynamics obtained from Autometrics for the credit equation[a]

Variable	Coefficient	t-value
$Constant$	-0.73	10.6
Δph_t	0.30	7.06
Δph_{t-4}	-0.12	2.64
Δy_{t-2}	-0.15	3.10
ΔE_{t-1}	-0.04	2.45
Δr_{t-3}	-0.24	2.34
ecm^{ph}_{t-1}	0.09	10.8
$CSeasonal_t$	-0.004	1.16
$CSeasonal_{t-1}$	-0.004	1.50
$CSeasonal_{t-2}$	-0.01	4.07
σ	0.009	
R^2	0.72	
$Adj.R^2$	0.69	
Diagnostics[b]	Test statistic	Value [p-value]
AR 1-5 test:	$F(5,76) =$	1.4959 [0.2011]
ARCH 1-4 test:	$F(4,83) =$	0.7501 [0.5608]
Normality test:	$\chi^2(2) =$	4.9864 [0.0826]
Hetero test:	$F(15,75) =$	0.8092 [0.6641]
Estimation Method	OLS (Autometrics with p-value = 0.05)	
Sample	1986q2-2008q4	

[a] Absolute t-values are reported.
[b] See Doornik and Hendry (2009a).

The results in Table 1 and Table 2 reveal some differences as compared to our preferred model. We note that both variables enter contemporaneously in both equations. Also, we observe that the income variable and the expectations variable are both highly significant in the credit equation with negative signs, which are not plausible *a priori* . Let us now turn to the two equations when they are estimated simultaneously to take care of potential endogeneity problems. Results are displayed in Table 3.

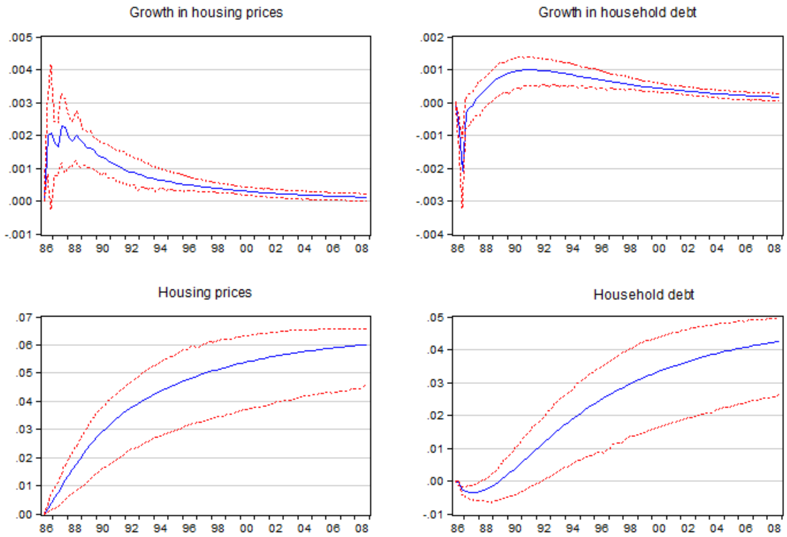
Table 3: System estimation of the specifications obtained by Autometrics (equation by equation)[a]

	Real housing prices		Real household debt	
Variable	Coefficient	t-value	Coefficient	t-value
Constant	1.00	3.78	−0.73	10.5
Δd_t	−0.26	0.49	–	–
Δph_t	–	–	0.32	5.50
Δph_{t-4}	0.36	3.65	−0.13	2.57
Δyh_{t-2}	–	–	−0.15	3.05
ΔE_t	0.12	3.88	–	–
ΔE_{t-1}	0.10	3.95	−0.04	2.48
ΔE_{t-2}	0.05	1.75	–	–
Δr_{t-3}	–	–	−0.24	2.37
Δr_{t-4}	−0.51	2.36	–	
Δt_{t-3}	0.06	2.50	–	
ECM^{ph}_{t-1}	−0.11	3.34	0.09	10.6
ECM^{d}_{t-1}	−0.10	3.85	–	–
Dummy, q1	−0.01	0.75	−0.005	1.26
Dummy, q2	−0.009	0.73	−0.004	1.55
Dummy, q3	−0.02	1.61	−0.01	4.07
Sargan	$\chi^2(43) =$	40.323 [0.5881]		
Log likelihood	567.99			
σ	0.016		0.0086	
Diagnostics[b]	Test statistic	Value [p-value]		
Vector SEM-AR 1-5 test:	$F(20, 138) =$	0.7944[0.7168]		
Vector Normality test:	$\chi^2(4) =$	4.7544[0.3134]		
Vector Hetero test:	$F(183, 81) =$	1.0260[0.4557]		
Estimation Method	FIML			
Sample	1986q2-2008q4			

[a] Absolute t-values are reported.
[b] See Doornik and Hendry (2009b).

The credit equation remains almost unaltered, while the housing price equation changes dramatically. First of all, the credit variable which is positive and highly significant in the single equation model has now changed sign and is insignificant. Also, the loadings have changed. As a final check of this model, we will explore how the implied dynamics of the system to a permanent increase in real disposable income would be. We follow exactly the same set up as in section 7.1 of the paper and the dynamic multipliers are graphed in Figure 1.

Figure 1: The alternative model: Dynamic multipliers of a 1 percent increase in real disposable household income.



Based on the dynamic multipliers from this alternative model, we see that it implies a negative response to household borrowing of an increase in income in the short run, which seems unreasonable from an economic point of view. Also, the credit effect on housing prices changes sign and turns out insignificant, though it was positive and highly significant in the single equation case. Furthermore, we observe relative big changes in the loadings in the housing price equation. On this background we conclude that this model is inferior to the one from the simultaneous model design reported in Table 5 in Section 6 of the paper.

4 Model without short run price homoegeneity

With reference to the forecasting exercise in Section 6 of the paper, this section discusses a version of the model, where we de-restrict the assumption of short run price homogeneity. To see whether the forecast failures for the credit growth in 2010q1 and 2011q1 (confer Figure 2 in the paper) may be due to the extremely cold winters, which lead to an extraordinary jump in electricity prices in each of the two quarters, we re-estimated the model for the case where short run price homogeneity is relaxed. As shown in the paper (see Figure 3), this improves the forecasting accuracy of the model – and in particular the credit forecasts. The estimation results underlying those forecasts are reported in Table 4.

We started by including the current and first lag of the change in the price deflator (Δpc) in both equations. However, these variables were only significant in the credit equation, and were therefore excluded from the housing price equation. As seen, the inclusion of Δpc_t and Δpc_{t-1} in the credit equation only has minor effects on the estimated parameters of the housing price equation, while the estimates of the credit equation are somewhat changed. That said, it seems to be changed for the better, since – as is evident from inspecting the table – derestricting short run price homogeneity improves the fit of the credit equation. Furthermore, both the current and lagged value are highly significant, and come with opposite signs. In fact, we can not reject the hypothesis that the two coefficients are equal in absolute value, i.e. suggesting that these terms are measuring a surprise inflation ($\Delta^2 pc_t = \Delta pc_t - \Delta pc_{t-1}$). This gives additional credence to our conjecture that the forecast failures in 2010q1 and 2011q1 are due to an unexpected increase in electricity prices.

Table 4: Short run dynamics [a]

	Real housing prices		Real household debt	
Variable	Coefficient	t-value	Coefficient	t-value
Constant	1.617	7.90	0.023	4.83
Δd_t	0.696	3.78	-	-
Δd_{t-1}	-	-	0.560	7.68
Δd_{t-3}	0.355	2.69	-	-
Δph_{t-4}	0.394	5.07	-	-
Δyh_{t-3}	-	-	0.084	1.99
ΔE_t	0.102	5.12	-	-
ΔE_{t-1}	0.100	4.76	-	-
ΔE_{t-2}	0.045	2.05	-	-
ΔR_{t-4}	-	-	-0.088	1.13
Δpc_t	-	-	-0.720	9.25
Δpc_{t-1}	-	-	0.528	5.89
ECM^{ph}_{t-1}	-0.172	7.86	-	-
ECM^{d}_{t-1}	-0.071	4.26	-0.025	4.63
Dummy, q1	0.025	3.87	-0.016	4.37
Dummy, q2	0.024	4.27	0.007	2.52
Dummy, q3	0.013	2.31	-0.019	7.36
Sargan		$\chi^2(48) =$	44.68 [0.6099]	
Log likelihood		603.68		
σ	0.0137		0.0064	
Diagnostics[b]		Test statistic	Value [p-value]	
Vector EGE-AR 1-5 test:		F(20,138)	0.50 [0.96]	
Vector Normality test:		$\chi^2(4)$	36.17 [0.00]	
Vector hetero test:		F(195,69)	0.67 [0.98]	
Estimation Method	FIML			
Sample	1986q2-2008q4 ($T = 91$)			

[a] Absolute t-values are reported.
[b] See Doornik and Hendry (2009b).

5 Additional tables not included in the paper

Table 5 reports the tests for lag reduction of the VAR(5) we started out with in Section 5 of the paper, while Table 6 reports ADF-tests for stationarity of the residuals in the short run model (confer Table 5 in the paper).

Table 5: Lag reduction for the exogenous variables in the unrestricted VAR [a],

Lags	log likelihood	SC	HQ	AIC
5	869.13433	-14.194	-15.824	-16.926
4	866.47195	-14.433	-15.964	-16.999
3	860.07987	-14.590	-16.022	-16.991
2	857.56754	-14.832	-16.166	-17.067
1	854.16023	-15.055	-16.290	**-17.124**
0	845.28489	**-15.157**	**-16.293**	-17.061
Tests of lag reduction				
5 to 4	F(6,112) =	0.55420 [0.7658]		
5 to 3	F(12,148) =	0.96638 [0.4836]		
5 to 2	F(18,158) =	0.83006 [0.6629]		
5 to 1	F(24,163) =	0.81618 [0.7127]		
5 to 0	F(30,165) =	1.0756 [0.3722]		
4 to 3	F(6,116) =	1.4069 [0.2178]		
4 to 2	F(12,153) =	0.98362 [0.4670]		
4 to 1	F(18,164) =	0.91767 [0.5582]		
4 to 0	F(24,168) =	1.2251 [0.2269]		
3 to 2	F(6,120) =	0.55985 [0.7615]		
3 to 1	F(12,159) =	0.66799 [0.7801]		
3 to 0	F(18,170) =	1.1519 [0.3071]		
2 to 1	F(6,124) =	0.78849 [0.5806]		
2 to 0	F(12,164) =	1.4710 [0.1398]		
1 to 0	F(6,128) =	2.1855[0.0485]*		
Estimation period:	1986q2-2008q4			

[a] Endogenous variables: Real housing prices, real household debt and real disposable income. Restricted variables: Real interest rate after tax, housing turnover, housing stock and a linear trend. Unrestricted variables: Constant and seasonal dummies.

References

Doornik, J. (2009). Autometrics. In J. L. Castle and N. Shepard (Eds.), *The Methodology and Practice of Econometrics*, pp. 88–121. Oxford University Press.

Doornik, J. A. and D. F. Hendry (2009a). *Empirical Econometric Modelling Using PcGive 13. Volume 1.* London: Timberlake Consultants.

Doornik, J. A. and D. F. Hendry (2009b). *Modelling Dynamic Systems Using PcGive 13. Volume 2.* London: Timberlake Consultants.

Table 6: Augmented Dickey-Fueller tests for structural residuals[a]

Levels					
Variable	t-ADF	5%-critical value	lags	trend	seasonal dummies
$\varepsilon_{\Delta ph}$	-8.846	-2.89	0	No	No
$\varepsilon_{\Delta d}$	-7.945	-2.89	1	No	No

[a] We only have data for ecm_d from 1985q1 because it includes the turnover. The residuals from the short run system is tested over the period 1988q3-2008q4 since we only obtain data for the error correction terms from 1986q2.

Harbo, I., S. Johansen, B. Nielsen, and A. Rahbek (1998). Asymptotic inference on cointegrating rank in partial systems. *Journal of Business and Economic Statistics 16*, 388 – 399.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control 12*, 231–254.

Johansen, S. (1992). Cointegration in partial systems and the efficiency of single-equation analysis. *Journal of Econometrics 52*, 389–402.